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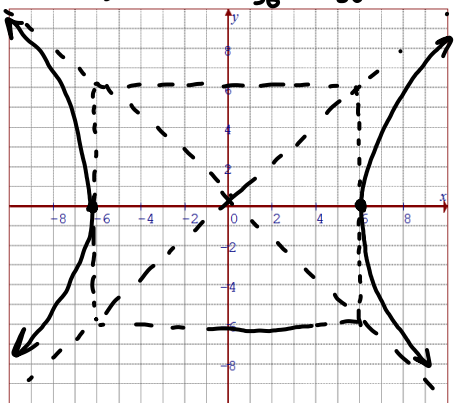
Date:_____

Math 10/11 Enriched: Section 7.3 Graphing Hyperbolas

1. Given each equation below, graph it on the grid provided:

a) $x^2 - y^2 = 36$ $\frac{x^2}{36} - \frac{y^2}{36} = 1$



Vertices: $(6,0), (-6,0)$ Asymptotes: $y = \pm 1$

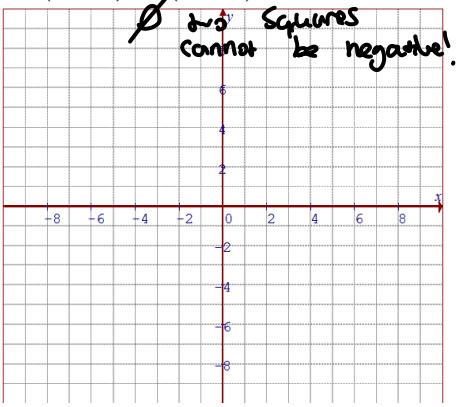
Length of Transverse: 12 Foci: $(\pm 36\sqrt{2}, 0)$

12

Domain: $x \geq 6$ or $x \leq -6$ Range: $y \in \mathbb{R}$

b) $(x+2)^2 + (y+2)^2 = -25$

$\emptyset$ two Squares cannot be negative!

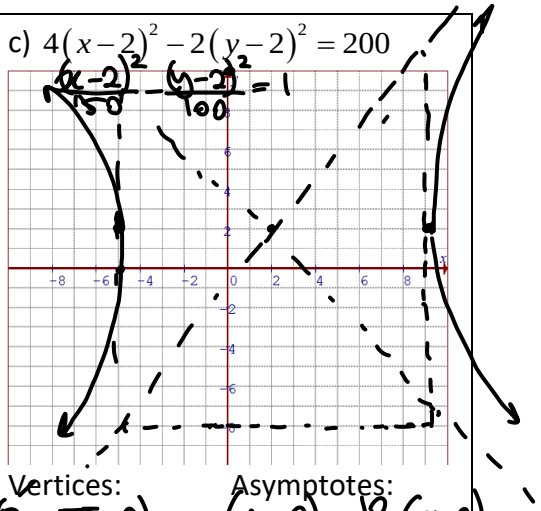


Vertices: Asymptotes:

Length of Transverse: $\emptyset$ Foci:

Domain: Range:

c) $4(x-2)^2 - 2(y-2)^2 = 200$ $\frac{(x-2)^2}{50} - \frac{(y-2)^2}{100} = 1$

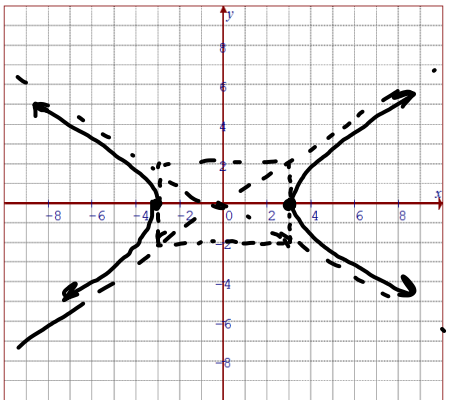


Vertices: $(2 \pm \sqrt{50}, 2)$ Asymptotes: $(y-2) = \frac{10}{\sqrt{50}}(x-2)$

Length of Transverse: $2\sqrt{50}$ Foci: $(2 \pm 5\sqrt{6})$

Domain: $x \leq 2 - \sqrt{50}$ or $x \geq 2 + \sqrt{50}$ Range: $y \in \mathbb{R}$

d) $-\frac{x^2}{9} + \frac{y^2}{4} = -1$

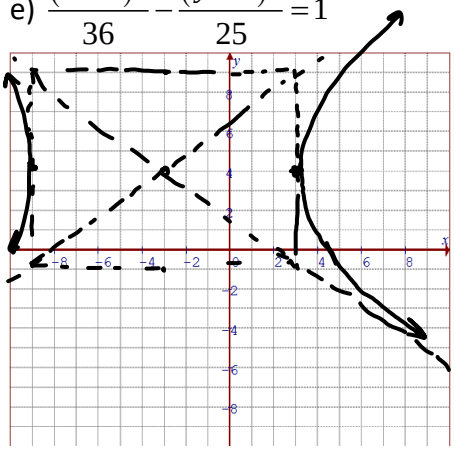


Vertices: $(\pm 3, 0)$ Asymptotes: $y = \pm \frac{2}{3}x$

Length of Transverse: 6 Foci: $(\pm\sqrt{13}, 0)$

Domain: $x \leq -3$ $x \geq 3$ Range: $y \in \mathbb{R}$

e) $\frac{(x+3)^2}{36} - \frac{(y-4)^2}{25} = 1$

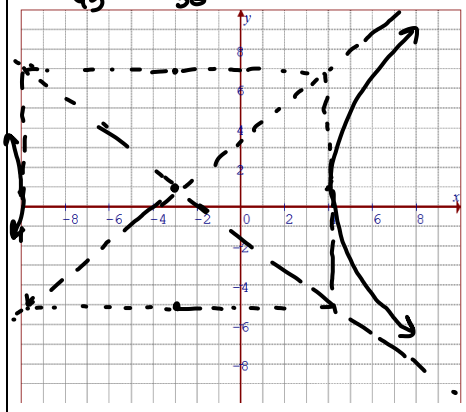


Vertices: $(3,4), (-9,4)$ Asymptotes: $(y-4) = \pm\frac{5}{6}(x+3)$

Length of Transverse: 12 Foci: $(-3 \pm \sqrt{61}, 4)$

Domain: $x \leq -9$ or $x \geq 3$ Range: $y \in \mathbb{R}$

f) $36(x+3)^2 - 49(y-1)^2 = 1764$ $\frac{(x+3)^2}{49} - \frac{(y-1)^2}{36} = 1$



Vertices: $(4,1), (-9,1)$ Asymptotes: $(y-1) = \pm\frac{6}{7}(x+3)$

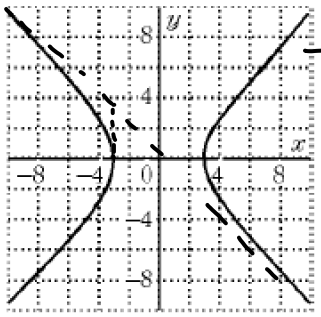
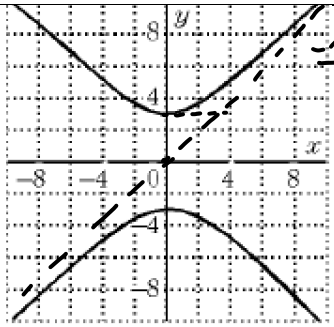
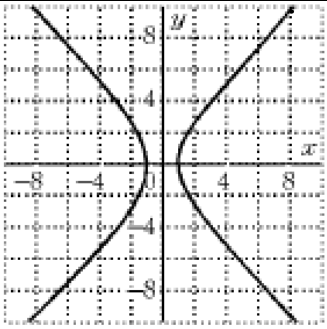
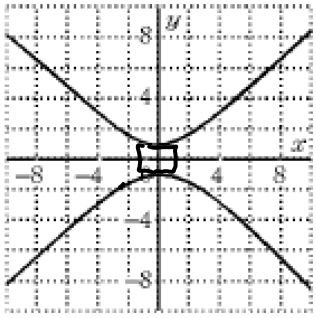
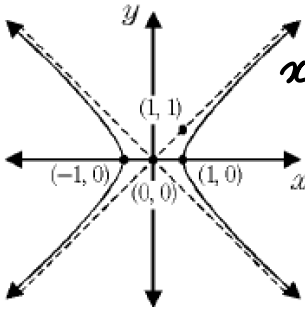
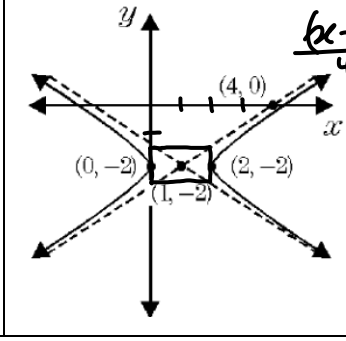
Length of Transverse: 14 Foci: $(-3 \pm \sqrt{85}, 1)$

Domain: $x \leq -9$ or $x \geq 4$ Range: $y \in \mathbb{R}$

2. Given each equation in general form, find the equation of the asymptote, location of the foci, and the equation in standard form:

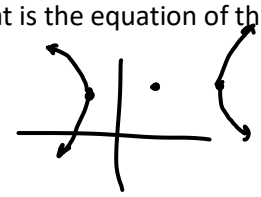
$2x^2-3y^2+4x-4=0$ $2(x+1)^2-3y^2=6$ $\frac{(x+1)^2}{3}-\frac{y^2}{2}=0$ $c=\sqrt{a^2+b^2}=\sqrt{5}$ foci: $(-1\pm\sqrt{5},0)$ Asymptote: $y=\frac{\sqrt{2}}{\sqrt{3}}(x+1)$	$4x^2-3y^2+8x-9y+16=0$ $4(x+1)^2-3(y+\frac{3}{2})^2=-\frac{75}{4}$ $\frac{(x+1)^2}{75}-\frac{(y+\frac{3}{2})^2}{\frac{225}{4}}=-1$ Asymptote: $(y+\frac{3}{2})=\pm\frac{\sqrt{3}}{2}(x+1)$ $c=\frac{5\sqrt{21}}{2}\Rightarrow$ foci: $(-1,-\frac{3}{2}\pm\frac{5\sqrt{21}}{2})$
$-9x^2+4y^2+54x+45=0$ $-9(x+3)^2+4y^2=36$ $\frac{-(x+3)^2}{4}+\frac{y^2}{9}=1$ Asymptote: $y=\frac{3}{2}(x+3)$ $c=\sqrt{4+9}=\sqrt{13}$ foci $=(-3,\pm\sqrt{13})$	$-4x^2+3y^2-12x-12y+11=0$ $-4(x+\frac{3}{2})^2+3(y-2)^2=-8$ $\frac{(x+\frac{3}{2})^2}{2}-\frac{(y-2)^2}{\frac{8}{3}}=1$ Asymptote: $(y-2)=\pm\frac{2}{\sqrt{3}}(x+\frac{3}{2})$

3. Given the diagram of each hyperbola, provide an equation that describes it:

 $\frac{y^2}{16}-\frac{x^2}{9}=1$	 $-\frac{x^2}{16}+\frac{y^2}{9}=1$	 $x^2-y^2=1$
 $-x^2+y^2=1$	 $x^2-y^2=1$	 $\frac{(x-1)^2}{4}-\frac{(y+2)^2}{\frac{1}{4}}=1$

4. A rectangular hyperbola has its centre at (3,5) and one of its vertex at (9,5). What is the equation of the hyperbola?

$\frac{(y-5)^2}{25} - \frac{(x-3)^2}{25} = 1$



5. Write the equation of the hyperbola with center at (5,3), vertex at (5,6), one asymptote with equation $4y - 3x = -3$ $y = \frac{3}{4}x - \frac{3}{4}$

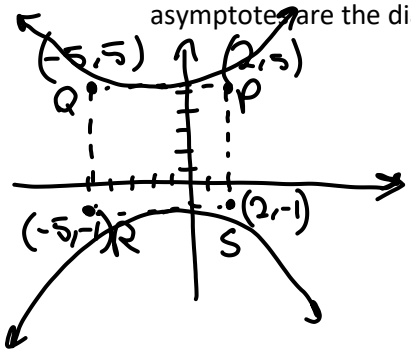
$\frac{(y-3)^2}{9} - \frac{(x-5)^2}{16} = -1$

6. What are the coordinates, in the form of (x,y) of the center of the hyperbola with equation $4x^2 - 2y^2 - 16x + 20y = 0$

$4(x-2)^2 - 2(y-5)^2 = -34$

$(x,y) = (2,5)$

7. Given rectangle PQRS, with P(2,5), Q(-5,5), S(2,-1), and R(-5,-1), find the equation of the hyperbola whose asymptotes are the diagonals and is tangent to sides PQ and RS.



$2a = 7$

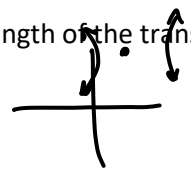
$a = \frac{7}{2}$

$b = 3$

$(h,k) = (-\frac{3}{2}, 2)$

$\frac{(y-2)^2}{9} - \frac{(x+\frac{3}{2})^2}{\frac{49}{4}} = 1$

8. State the coordinates of the vertices, the length of the transverse axis, and the equations of the asymptotes of the hyperbola: $\frac{(x-3)^2}{16} - \frac{(y-6)^2}{9} = 1$



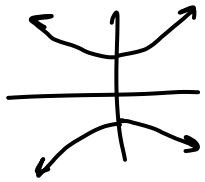
Vertices: $(3 \pm 4, 6)$

Transverse axis = 6

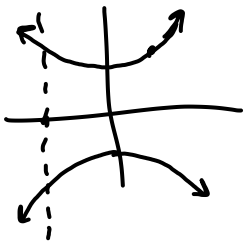
Asymptote: $y + 6 = \frac{4}{3}(x+3)$

9. The vertices of a hyperbola are (2,3) and (2,-5). If one of the asymptotes has a slope of 2/3, determine an equation for the hyperbola:

$$\frac{(x-2)^2}{4} - \frac{(y-1)}{9} = -1$$



10. A rectangular hyperbola of the form $y^2 - x^2 = m^2$ has points (3,5) and (-5,z) on the graph. Determine the value of "z"



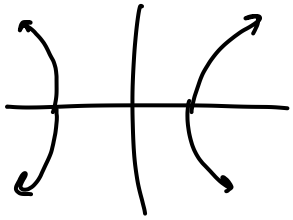
$25 - 9 = m^2$

$z^2 - 25 = m^2$

$z^2 - 25 - 25 + 9 = 0$

$z^2 = 41 \Rightarrow z = \pm\sqrt{41}$

11. Given an equation in the form of $Ax^2 + By^2 = 1$, which value "A" or "B" must be negative in order for it to be an hyperbola with its vertices on the x-axis?



B has to be negative

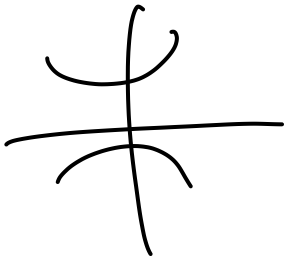
12. Determine the equations of a hyperbola with asymptotes $y = \pm\frac{4}{3}x + 1$ and the distance between its vertices is 8 units long.

$2b = 8 \Rightarrow b = 4$

$a = 3$

$y = \pm\frac{4}{3}\left(x - \frac{3}{4}\right)$

centre $= \left(\frac{3}{4}, 0\right)$



equation: $\frac{\left(x - \frac{3}{4}\right)^2}{9} - \frac{y^2}{16} = 1$

13. A point where both coordinates are integers is known as a lattice point. How many lattice points lie on the parabola: $x^2 - y^2 = 2000^2$

$(x-y)(x+y) = 2000^2$

2 2000000

$(x-y) + (x+y) = 2x = 2000000$

sum of factors has to be even.

either both factors are even or odd. (Both odd is not possible here though).

$\frac{2x}{2^6 \times 5^6 \text{ left}} \times \frac{2x}{} = 2000^2 = 2^8 \times 5^6$

49 pairs.

we can swap places, so

$49 \cdot 2 = \boxed{98}$

14. Considering the equation $Ax^2 + By^2 + C = 0$. What coordinates must be satisfied by A, B, and C for this equation to represent each of the following conics:

    a. A circle with centre at the origin

    b. A rectangular hyperbola with the centre at the origin and vertices on the X-axis

    c. A rectangular hyperbola with the centre at the origin and vertices on the Y-axis

4. Given rectangle $PQRS$, with $P(2,5)$, $Q(-5,5)$, $S(2,-1)$, and $R(-5,-1)$, find the equation of the hyperbola whose asymptotes are the diagonals of the rectangle and is tangent to sides PQ and RS.

2. $\frac{(y-2)^2}{9} - \frac{(x+\frac{3}{2})^2}{\frac{49}{4}} = 1$

3. Sketch the graphs of these relations on the same grid if possible:

    a) $x^2 + y^2 = 9$ b) $x^2 + y^2 = 0$ c) $x^2 + y^2 = -9$

4.